If (x_n) is convergent, then in particular it is Cauchy and the direct implication is obvious.

Reciprocally, let $\epsilon > 0$. There is some $N \in \mathbf{N}, \overline{\lim}_{m \to +\infty} |x_N - x_m| \leq \epsilon$. Now there is $M \in \mathbf{N}, \forall m, m' \geq M, \ |x_N - x_m| \leq 2\epsilon$. Eventually, $\forall m, m' \geq M, |x_m - x_{m'}| \leq |x_N - x_m| + |x_N - x_{m'}| \leq 4\epsilon$. This proves that (x_n) is Cauchy in $\mathbf{R}$, whence it is convergent.