Integrate by parts as follows:

$\int_0^{2\pi} f(x)cos(x)dx =$
$[f(x)sin(x)]_0^{2\pi} - \int_0^{2\pi} f'(x)sin(x)dx =$
$-([-f'(x)cos(x)]_0^{2\pi} - \int_0^{2\pi} -f''(x)cos(x)dx) =$
$\int_0^{2\pi} f''(x)(1 - cos(x))dx \geq 0$, since $f'' \geq 0$ by convexity of f.