One of your subintervals must contain 0, say $I = [0, a_1]$, where $a_1 < 1$. Similarly, one of them must contain 1, say $J = [a_2, 1]$, where $a_2 > a_1$, since the intervals shall not overlap. Now take a point $x \in ]a_1, a_2[$. $x$ must be contained in a certain interval $K = [a_3, a_4]$, where $a_1 < a_3 \leq a_4 < a_2$. Clearly enough, the set of those intervals created by repeating indefinitely the above process in-between the previously created intervals is idempotent to $\mathbf{R}$.