By the least upper bound axiom, the *sup* exists since the given set has b as an upper bound, and is nonempty since a belongs to it. Now by continuity of f, $f(sup(\{x \in [a,b] : f(x) \leq \gamma\}))$ cannot be $< \gamma$, otherwise you can find $\epsilon > 0$ such that $f(sup(\{x \in [a,b] : f(x) \leq \gamma\}) + \epsilon) \leq \gamma$, which contradicts the definition of the *sup*. It cannot be $> \gamma$ either (why ?). This solves the question.

Remember that usually the previous theorem is proved by saying that the continuous image of a connected space is connected.